interpretations of molecular forces if the field for a polar gas is replaced by an effective symmetrical field. On the one hand, this is confirmed by our calculations for $\eta_0(T)$ for water vapor, including those with the molecular potentials of [6], which were derived from the p — V — T data (curve 7 of Fig. 1). The deviations from (1) are up to 3-5%, and they increase toward high and low temperatures. On the other hand, these deviations are comparatively small, so one can say that one gets reasonably satisfactory results within the framework of the Chapman — Enskog theory by using various types of experimental data with a reasonably realistic potential.

NOTATION

T, temperature; $\eta_0(T)$, zero-density dynamic viscosity; $T_{cr} = 647.27°K$; a_i, interpolation parameters for viscosity; $T^* = kT/\varepsilon$, reduced temperature; $W_j = 1/(\Delta\eta_{0j})^2$, statistical weight; $\Delta\eta_{0j} = \delta_{\eta_{0j}}\eta_{0j}$, absolute error; $\delta_{\eta_{0j}}$, relative error; σ, ε/k, μ, δ, α, parameters of potentials; $\Omega^{(2.2)^*}(T^*)$, reduced collision integral; b_i, parameters of interpolation formula for collision integrals $\Omega^{(2.2)^*}(T^*)$; $r^* = r/\sigma$, reduced internuclear distance; $\delta^* = \mu^2/[2(\varepsilon/k)_0\sigma_0]$, reduced dipole moment; $\gamma^* = 2a/\sigma_0$; $2a$, diameter of spherical core; $\delta^{**} = \delta^* G(\theta_1, \theta_2, \Phi)$; $G(\theta_1, \theta_2, \Phi) = 2\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\Phi$, function incorporating the effects of dipole orientation; $T_0^* = T/(\varepsilon/k)_0$, reduced temperature; $\alpha^* = \alpha/\sigma_0^3$, reduced molecular polarizability.

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ACCURACY OF A ONE-DIMENSIONAL APPROXIMATION FOR DOUBLE RODS

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Limits of applicability of one-dimensional models in computing temperatures in two-layered rods are established on the basis of comparing the one-dimensional approximation with the exact two-dimensional solution.

To solve heat-conduction problems in cylindrical armature elements of constant and variable cross section, a one-dimensional approximation method is used. The one-dimensional approximation yields satisfactory results for homogeneous rods with low values of the Biot criterion [1]. The limits of a possible application of this method were established in [2] in an example of a solid homogeneous cylinder. Meanwhile, strict criteria for double rods consisting of heterogeneous materials are completely absent.

In order to establish admissible quantitative limits for the applicability of the one-dimensional approximation method for double rods, let us consider a cylinder consisting of heterogeneous materials with the coefficients of thermal conductivity λ_1 and λ_2. The cylinder is heated at the base and exchanges heat with the surrounding medium of temperature T_m via the side surface because of convection and radiation with a total constant coefficient of heat emission α. The end-face surface is considered heat-insulated, which is the

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TABLE 1. Roots μ_n and Values μ_1^* for Several Values of the Biot Criterion for k_a = 0.5

	$k_\lambda = 0,1$	$k_\lambda = 1,0$	$k_\lambda = 10,0$
Bi = 0,01	0,16051 4,62023 6,52822 10,86038 12,77635 17,13139 19,04629	0,14124 3,83432 7,01701 10,17445 13,32444 16,47124 19,61637	0,07824 2,95932 7,52153 9,47851 13,87785 15,80746 20,18906
	μ_1^*=0,16063	μ_1^*=0,14142	μ_1^*=0,07845
Bi = 0,05	0,35769 4,62253 6,53856 10,86142 12,78169 17,13206 19,04988	0,31426 3,84473 7,02271 10,17838 13,32744 16,47366 19,61841	0,17313 2,98448 7,52304 9,48581 13,87868 15,81180 20,18963
	μ_1^*=0,35921	μ_1^*=0,31623	μ_1^*=0,17540
Bi = 0,1	0,50371 4,62537 6,55146 10,86272 12,78836 17,13289 19,05436	0,44168 3,85771 7,02982 10,18329 13,33120 16,47670 19,62096	0,24172 3,01553 7,52493 9,49493 13,87971 15,81724 20,19034
	μ_1^*=0,50799	μ_1^*=0,44721	μ_1^*=0,24807

characteristic mode of operation of a different kind of edge in the armature. The stationary dimensionless temperature u will be determined by the Laplace equation

$$\Delta u_i(r,\ z) = 0 \quad (i = 1,\ 2) \tag{1}$$

and the boundary conditions

$$u_i(r,\ 0) = 1,\ u_{iz}(r,\ l) = 0 \quad (i = 1,\ 2),\ u_{2r}(a,\ z) + \frac{\alpha}{\lambda_2}\, u_2(a,\ z) = 0. \tag{2}$$

The merger conditions are assured by equality of the temperatures and heat fluxes on the boundary of the inner and outer cylinders

$$u_1(r_1,\ z) = u_2(r_1,\ z), \quad \lambda_1 u_{1r}(r_1,\ z) = \lambda_2 u_{2r}(r_1,\ z). \tag{3}$$

The problem (1)-(3) mentioned can be solved by separation of variables

$$u(r,\ z) = \sum_{n=1}^{\infty} A_n \frac{\operatorname{ch}\mu_n \dfrac{l-z}{a}}{\operatorname{ch}\mu_n \dfrac{l}{a}} R_n(r), \tag{4}$$

where μ_n are the roots of the characteristic equation

$$\frac{\mu}{\mathrm{Bi}} = \frac{[k_\lambda J_1(k_a\mu)Y_0(k_a\mu) - J_0(k_a\mu)Y_1(k_a\mu)]\,J_0(\mu) - J_0(k_a\mu)}{[k_\lambda J_1(k_a\mu)Y_0(k_a\mu) - J_0(k_a\mu)Y_1(k_a\mu)]\,J_1(\mu) - J_0(k_a\mu)} \to \frac{J_1(k_a\mu)(k_\lambda - 1)Y_0(\mu)}{J_1(k_a\mu)(k_\lambda - 1)Y_1(\mu)}, \tag{5}$$

$$R_n(r) = \begin{cases} J_0(\mu_n r/a), & 0 < r < r_1, \\ J_0(k_a\mu_n)\dfrac{J_0(\mu_n r/a) - \omega_n Y_0(\mu_n r/a)}{J_0(k_a\mu_n) - \omega_n Y_0(k_a\mu_n)}, & r_1 < r < a, \end{cases} \tag{6}$$

$$\omega_n = [\mathrm{Bi}\,J_0(\mu_n) - \mu_n J_1(\mu_n)]/[\mathrm{Bi}\,Y_0(\mu_n) - \mu_n Y_1(\mu_n)].$$

Using the usual method, it can be shown that the eigenfunctions of the problem $R_n(r)$ are orthogonal in the interval $[0,\ a]$ with the weight

$$q(r) = \begin{cases} k_\lambda r, & 0 < r < r_1, \\ r, & r_1 < r < a \end{cases}$$

and the heat amplitudes A_n are given by the expression

$$A_n = \frac{2\,\mathrm{Bi}\,R_n(a)/\mu_n^2}{k_a^2(k_\lambda - 1)[J_0^2(k_a\mu_n) - k_\lambda J_1^2(k_a\mu_n)] + (1 + \mathrm{Bi}^2/\mu_n^2)\,R_n^2(a)}. \tag{7}$$

To determine the temperatures by means of (4)-(7) it is necessary to know the roots μ_n of the transcendental equation (5). Numerical values of the roots μ_n are presented in the table for three characteristic values of k_λ computed on an "M-220" type electronic computer. Some values of the heat amplitudes and of the quantities ω_n are presented in [3].

In the one-dimensional case, the problem of heat transfer in a double rod in the case when the thermophysical characteristics are given functions of the temperature reduce to the solution of the nonlinear differential equation [1,4]

$$\frac{d}{dz}\left[(\lambda_1 f_1 + \lambda_2 f_2)\frac{dT}{dz}\right] = \alpha\chi_2(T - T_m) + \sigma\varepsilon\chi_2\left[\left(\frac{T}{100}\right)^4 - \left(\frac{T_m}{100}\right)^4\right], \tag{8}$$

where f_1 and f_2 are the respective cross-sectional areas for the inner and outer cylinders, and χ_2 is the outer perimeter of the cross section. For a step double rod, (8) retains its form for each step, and the conditions of equality of the temperatures and heat fluxes in the plane separating the steps are added in the case of an ideal thermal contact. In the case of a double circular rod, i.e., for $f_1 = \pi r_1^2$, $f_2 = \pi(a^2 - r_1^2)$, $\chi_2 = 2\pi a$, the one-dimensional temperature distribution under appropriate simplifying assumptions relative to the thermophysical characteristics and the radiation heat exchange will satisfy (8), which is rewritten as

$$\frac{d^2 u_{\mathrm{one}}}{dz^2} - \frac{2\,\mathrm{Bi}}{[1 + k_a^2(k_\lambda - 1)]\,a^2}\,u_{\mathrm{one}} = 0 \tag{9}$$

with boundary conditions of the form (2).

For long rods of modern structural materials with heat exchange through the side surface because of free convection and radiation, the criterion is $\mathrm{Bi} \ll 1$. Using the relations for Bessel functions of the first and second kinds with small values of the arguments [5], the value of the first root of the characteristic equation (5) μ_1^* can be obtained taking account of the smallness of the criterion Bi:

$$\mu_1^{*2} = 2\,\mathrm{Bi}/[1 + k_a^2(k_\lambda - 1)]. \tag{10}$$

Several values of the roots μ_1^*, computed by means of (1) for characteristic values of k_λ and B_i for $k_a = 0.5$, are represented in Table 1. The values of μ_1^* hence turn out to be close to the exact values of the first root of the characteristic equation (5), which permits successful utilization of (10). Knowledge of the first root of an equation of the type (5) turns out to be especially important in the case of the appropriate nonstationary problem, since this root governs the temperature change of the rod in the regular thermal mode.

In the case of small Bi $A_1 \to 1$, $A_n \to 0$ ($n = 2, 3, \ldots$) and $R_1(r) \to 1$ so that the solution of the two-dimensional problem (4) acquires for small Bi the form

$$u(r, z) = \mathrm{ch}\,\mu_1^*\,\frac{l - z}{a} \Big/ \mathrm{ch}\,\mu_1^*\,\frac{l}{a}, \tag{11}$$

which is exactly the solution given by the one-dimensional approximation (9) with conditions of the type (2). Therefore, it becomes evident that the one-dimensional solution results explicitly from the solution of the two-dimensional problem for a double rod with small values of the Biot criterion.

By using the data in the table, the influence of the radius on the temperature distribution for small Bi can be established, the temperatures obtained by using the one-dimensional approximation (11) and in the solution of the appropriate two-dimensional problem (4) can be compared, and strict quantitative limits for the applicability of the one-dimensional approximation method can also be set up. The comparison is carried out for the surface temperature $u(a, z)$ and the temperature $u(0, z)$ on the axis of the double cylinder. Graphs of the dependences $u|_{r=a} - u_{\mathrm{one}}$ and $u_{\mathrm{one}} - u|_{r=0}$ are represented in the Fig. 1 for three characteristic values of k_λ and Bi = 0.05. An analysis of these dependences shows that the absolute error of the one-dimensional approximation does not exceed $0.023(T_0 - T_m)$ for the most unfavorable case in k_λ ($k_\lambda = 10.0$). The absolute error of the one-dimensional approximation gradually increases with the increase in the number Bi and reaches the quantity $0.078(T_0 - T_m)$ for Bi = 0.2 and $k_\lambda = 10.0$. The values of the relative errors are hence 2.9 and 17.5%, respectively.

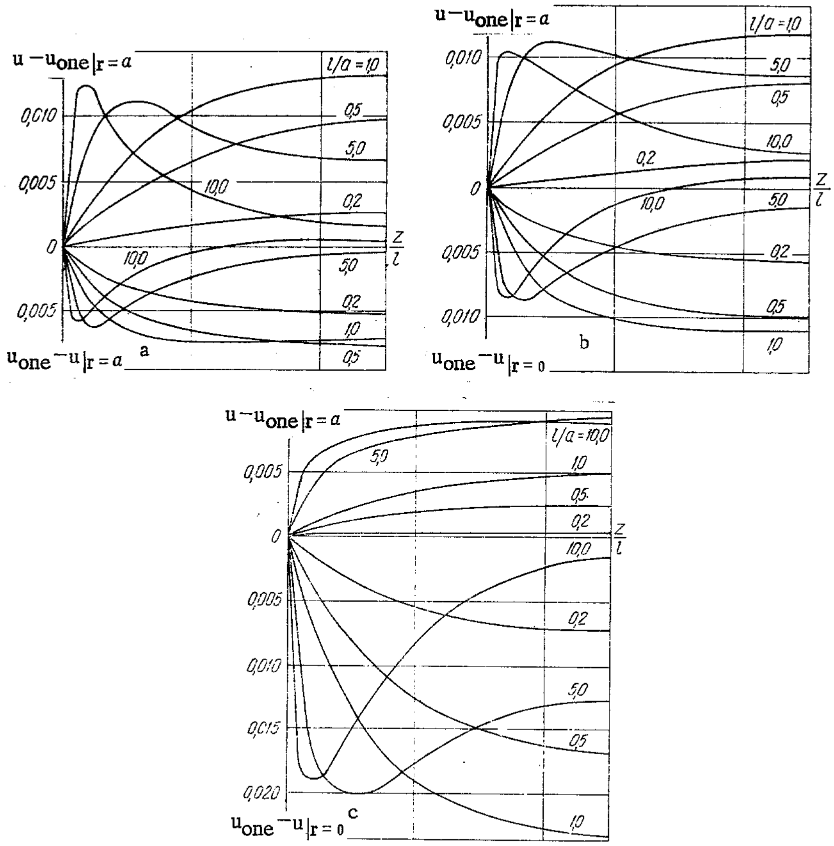

Fig. 1. Comparison between the one-dimensional approximation and the exact two-dimensional solution for three characteristic values of k_λ : a) $k_\lambda = 0.1$; b) 1.0; c) 10.0 for $k_a = 0.5$ and Bi=0.05.

An important characteristic for many applied problems is the ratio between the temperature of the double rod and the temperature of a homogeneous rod at corresponding sections. This characteristic is called the temperature efficiency [6]. The tabular data permit determination of the temperature efficiency of the double rods under consideration. Thus, the temperature efficiency increases from 0.84 to 1.15 for Bi = 0.05 and k_λ varying between 0.1 and 10.0.

Computations for double rods in the presence of a contact resistance between the inner and outer cylinders indicate the lack of its influence of the temperature distribution for small Bi.

It must be noted that the greatest deviations from the one-dimensional solution are observed near the base, especially in the case of long cylinders ($l/a = 5.0$ and $l/a = 10.0$), which permits representation of the temperature field structure in a first approximation as a quasi-one-dimensional solution with a boundary-layer correction (in the Vishik — Lyusternik sense) at this base [3]. This permits taking account of the dependence of the temperature on the transverse coordinate and a substantial diminution in the error of the one-dimensional model.

Meanwhile, the mentioned quasi-one-dimensional formulas with a small correction along the radius can also be obtained without turning to the exact solution (4) by direct asymptotic integration of the initial multidimensional problem, by using singular perturbation methods [7], for example.

NOTATION

T, absolute temperature; z, coordinate along the rod length; r, radial coordinate; l, linear dimension; α, coefficient of heat emission; ε, emissivity; σ, Stefan — Boltzmann constant; T_0 = const, temperature of the lower base; $u = (T - T_m)/(T_0 - T_m)$, dimensionless temperature; μ_1^*, approximate value of the root μ_1; $J_0(x)$, $J_1(x)$, $Y_0(x)$, $Y_1(x)$, Bessel functions of the first and second kinds; $Bi = \alpha a/\lambda_2$, Biot criterion; $k_\lambda = \lambda_1/\lambda_2$, $k_a = r_1/a$, dimensionless coefficients. Indices: m, surrounding medium; 1, inner cylinder; 2, outer cylinder; n, appropriate member of the series.

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TEMPERATURE CONDITIONS OF ROCK EXCAVATION

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Heat exchange is considered between ventilating air and mining rocks in the case of variable air temperature at the rock excavation entrance. Formulas are given for the temperature of the ventilating air according to the extent of exploitation.

In developing deep pits with seams of high temperature as well as in many pits in the Far North that have seams of low temperature the need arises for regulating the temperature of the ventilating air. The problem thus arises of determining the temperature of the ventilating air along the length of the underground excavation at different time instants. An exact solution of this problem which can be obtained by using the operational calculus is very cumbersome. Several hours of machine time are needed to set it on an M-220 electronic computer. Numerous approximation methods have been proposed to find the solution. It was proposed in [1] that the nonstationary heat exchange be taken into account between the air and the mined rocks with the aid of a coefficient of nonstationary heat exchange; to determine the latter a dependence was assumed which was an approximation to the exact solution. In [2] the formula for the nonstationary exchange coefficient was obtained by approximating the solution of the problem under consideration on a hydrointegrator. In the present article the integral method [3] is used to solve the heat-exchange problem between the ventilating air and the mined rocks which, as shown below, produces a good agreement with the exact solution. The solution is obtained for the case of variable air temperature at the mining operation entrance.

The equation of the heat flow of the ventilating air at the production face is given by

$$\rho_a c_a v \frac{\partial T}{\partial z} = \frac{2}{r_0} \lambda \frac{\partial \theta}{\partial \xi}\bigg|_{\xi=r_0} + \frac{q}{\pi r_0^2}, \tag{1}$$

$$T|_{z=0} = T_0(t). \tag{2}$$

In the above, q denotes the power of the internal heat sources per one meter of output face. Here heat emission due to moisture condensation or to various operating devices, etc., can also be included.

The heat-conduction equation for the rock mass surrounding the face is

$$\frac{\partial \theta}{\partial t} = a\left(\frac{1}{\xi}\cdot\frac{\partial \theta}{\partial \xi} + \frac{\partial^2 \theta}{\partial \xi^2}\right), \tag{3}$$

$$\theta|_{\tau=0} = \theta_M,$$

$$\lambda \frac{\partial \theta}{\partial \xi}\bigg|_{\xi=r_0} = \left(\frac{1}{\alpha} + \sum_n \frac{\delta_n}{\lambda_n}\right)^{-1} (\theta|_{\xi=r_0} - T). \tag{4}$$

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